

# NEUTRINO MASS AS A CONSEQUENCE OF THE EXACT SOLUTION OF 3-3-1 GAUGE MODELS WITHOUT EXOTIC ELECTRIC CHARGES

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## Abstract

The unjustly neglected method of exactly solving generalized electro-weak models - with an original spontaneous symmetry breaking mechanism based on the gauge group  $SU(n)_L \otimes U(1)_Y$  - is applied here to a particular class of chiral 3-3-1 models. This procedure enables us - without resorting to any approximation - to express the boson mass spectrum and charges of the particles involved therein as a straightforward consequence of both a proper parametrization of the Higgs sector and a new generalized Weinberg transformation. We prove that the resulting values can be accommodated to the experimental ones just by tuning a sole parameter. Furthermore, if we take into consideration both the left-handed and right-handed components of the neutrino (included in a lepton triplet along with their corresponding left-handed charged partner) then we are in the position to propose an original method for the neutrino to acquire a very small but non-zero mass without spoiling the previously achieved results in the exact solution of the model. In order to be compatible with the existing phenomenological data, the range of that sole parameter imposes in our method a large order of magnitude for the VEV  $\langle \phi \rangle \sim 10^6$  TeV. Consequently, the new bosons of the model have to be very massive.

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## 1 Introduction

In Ref. [1] devoted to exactly solving generalized chiral electro-weak  $SU(n)_L \otimes U(1)_Y$  gauge models, a new Higgs mechanism has been proposed that acts as a good mathematical strategy able to break the symmetry of the model up to the universal residual one  $U(1)_{em}$ , as in the Standard Model (SM). A non-degenerated boson mass spectrum is therefore expected once the spontaneous symmetry breaking (SSB) is accomplished. This could be achieved by introducing an adequate metric into the kinetic term of the

scalar sector of the model. Then, based on a new and generalized Weinberg transformation (gWt), we can separate the neutral gauge fields keeping at the same time massless the electromagnetic one. Furthermore, one can give the particles their charges (both electric and neutral ones) by identifying the coupling coefficients of certain currents in the general method. Fermion masses arise from special Yukawa couplings consisting of tensor products among certain Higgs multiplets that get the usual form of Yukawa couplings when boosted towards unitary gauge.

For these models the renormalizability criteria are also satisfied:

- *The axial anomaly cancellation* - that can in turn explain the number of fermionic generations in concrete models - has a specific formulation (see Sec. 6.2. in Ref. [1]).
- *The formal dimension of terms in the new Yukawa couplings* - by means of which one controls the divergences that could appear in the propagator approach - is maintained at a suitable level (see Sec. 4.3. in Ref [1]).

Reference [1] ends up with the successful confirmation of the general method in the particular case of the Pisano, Pleitez and Frampton (PPF) model. Designed to explain the electro-weak interaction based on the local gauge group  $SU(3)_c \otimes SU(3)_L \otimes U(1)_Y$ , this was the first of the so called "3-3-1 models" but not the only one. The PPF model (Refs. [2, 3]) had to pay the price of dealing with exotic electric charges for some quarks ( $\pm 5e/3$ , and  $\pm 4e/3$  respectively) as doubly charged bileptons were allowed to occur in the model. Concerning the gauge sector, it is obvious that for algebraic reasons a new neutral physical gauge boson (besides the one coming from SM) has to occur. Due to it, a new neutral charge is added to the "old" charges from SM in order to fully describe the fermion particles in the model. In order to improve the particle content of 3-3-1 models, new family representations were proposed as candidates for the fermion sector. Therefore, some new 3-3-1 models that avoid exotic electric charges have emerged in the literature [4] - [8]. Regarding the manner in which those models accomplish the anomaly cancellation requirement, they can be either "family models" (with generations treated differently) or "one-family models" (with generations obtained by replication). There are some papers [9] - [13] that carry a systematic analysis of those possible models, keeping alive the anomaly cancellation principle in each particular case and even predicting new kinds of models by combining the anomaly factors so that they vanish by an interplay.

In the following, we assume the results of the general method in Ref. [1] as valuable and useable in concrete cases and apply them to a particular class of "no-exotic-electric-charge" 3-3-1 models (namely, models C and D in Ref. [9]) attempting to find its exact solution. Finally, the correlation with experimental data (Ref. [14]) can be made just by tuning certain parameters of the model. At the same time we show that this procedure allows us to propose a mechanism for the neutrino to acquire a very small - *but non-zero!* - Majorana mass when the breaking scale of the model reaches a very large order of magnitude.

The paper is organized as follows. The main results of the general method are briefly presented in Section 2, as they stand as the starting point to our results. Section 3 deals with the exact solution of a particular class of 3-3-1 models. All the SM

phenomenology is reproduced. The boson masses and the charges of the particles are obtained just by making an appropriate parameter choice in the general method. This leads straightforwardly to the spinor sector's content of the models C and D from Ref. [9]. Then we present the Yukawa terms that enable fermion families to acquire their masses. Section 4 focuses on the neutrino mass issue by introducing a symmetric matrix with sextet transformation involving tensor products of Higgs multiplets. An appropriate way to embed neutrino mixing in our method is also shown with special emphasis on its compatibility with experimentally obtained mass squared differences. In the concluding remarks the resulting neutrino mass range and its implications for the exact solution of 3-3-1 model are discussed.

## 2 Preliminaries

Let us review the main results of exactly solving the generalized  $SU(n)_L \otimes U(1)_Y$  gauge model as they come from Ref. [1]. In accordance with the general method, the scalar sector of any "pure left" gauge model must consist of  $n$  Higgs multiplets  $\phi^{(1)}, \phi^{(2)}, \dots, \phi^{(n)}$  satisfying the orthogonal condition  $\phi^{(i)+}\phi^{(j)} = \phi^2\delta_{ij}$  in order to eliminate unwanted Goldstone bosons after SSB. Here  $\phi$  is a gauge-invariant real field variable and  $n$  is the dimension of the fundamental irreducible representation of the gauge group. The parameter matrix  $\eta = (\eta_0, \eta^{(1)}, \eta^{(2)}, \dots, \eta^{(n)})$  with the property  $Tr\eta^2 = 1 - \eta_0^2$  is also introduced. Then, the Higgs Lagrangean density (Ld) stands:

$$\mathcal{L}_H = \frac{1}{2}\eta_0^2\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\sum_{i=1}^n\left(\eta^{(i)}\right)^2\left(D_\mu\phi^{(i)}\right)^+\left(D^\mu\phi^{(i)}\right) - V(\phi) \quad (1)$$

where  $D_\mu\phi^{(i)} = \partial_\mu\phi^{(i)} - ig(A_\mu + y^{(i)}A_\mu^0)\phi^{(i)}$  means covariant derivatives of the model. After the SSB the boson masses take the forms:

$$M_i^j = \frac{1}{2}g\langle\phi\rangle\sqrt{\left[\left(\eta^{(i)}\right)^2 + \left(\eta^{(j)}\right)^2\right]} \quad (2)$$

for the non-diagonal gauge bosons (which usually are charged but, as we can easily observe in the 3-3-1 models under consideration in the next section, one of them could also be neutral) and:

$$(M^2)_{ij} = \langle\phi\rangle^2 Tr(B_i B_j) \quad (3)$$

with:

$$B_i = g\left[D_i + \nu_i(D\nu)\frac{1 - \cos\theta}{\cos\theta}\right]\eta \quad (4)$$

for the diagonal gauge bosons of the model. The last ones are neutral without exception. The  $\eta$  parameter diagonal matrix comes from the scalar sector and it will essentially determine the mass spectrum of the model,  $\theta$  is the rotation angle around the versor  $\nu$  orthogonal to the electromagnetic direction in the parameter space [1]. The versor condition holds  $\nu_i\nu^i = 1$ . For the concrete models we will work on,  $D$ s are the

Hermitian diagonal generators (Cartan subalgebra) of the  $SU(3)_L$  group, *i.e.*  $D_1 = T_3$  and  $D_2 = T_8$  connected to the Gell-Mann matrices in the manner  $T_a = \lambda_a/2$ .

After we extracted the electromagnetic potential (the field  $A_\mu^{em}$  that is massless), a special  $SO(n-1)$  transformation  $\omega$  remains to be determined in each particular case, in order to bring the mass matrix into the physical basis  $(A_\mu^{em}, Z_\mu, Z'_\mu)$  so that the masses of physical neutral bosons are just the eigenvalues of the diagonal form of the matrix (3). The gWt can be defined as:

$$A_\mu^0 = A_\mu^{em} \cos \theta - \nu_i \omega_{\cdot j}^i Z_\mu^j \sin \theta \quad (5)$$

$$A_\mu^k = \nu^k A_\mu^{em} \sin \theta + [\delta_i^k - \nu^k \nu_i (1 - \cos \theta)] \omega_{\cdot j}^i Z_\mu^j \quad (6)$$

Moreover, the charges of the particles can be identified as coupling coefficients of the currents. In a certain representation  $\rho$  the charge operators have the following diagonal forms [1]:

$$Q^\rho(A_\mu^{em}) = g [(D^\rho \nu) \sin \theta + y_\rho \cos \theta] \quad (7)$$

$$Q^\rho(Z_\mu^i) = g [D_k^\rho - \nu_k (D^\rho \nu) (1 - \cos \theta) - y_\rho \nu_k \sin \theta] \omega_{\cdot i}^{k\cdot} \quad (8)$$

>From Eq. (7) one can write the following formula for the fundamental multiplet ( $y_\rho = 0$ ):

$$g^2 \sin^2 \theta = 2Tr(Q^2) \quad (9)$$

All the charges of the particles in a certain multiplet can be obtained straightforwardly from this point just by taking into consideration the multiplet's own representation  $\rho$  with its hypercharge  $y_\rho$  value.

### 3 The Exact Solution of a Special Class of 3-3-1 Models

The general method can be accommodated to PPF model's fermionic content (see Sec. 7 in Ref. [1]) or to any other one's just by choosing a particular set of versors in gWt which will lead to the desired electric charges of the fundamental multiplet. In our paper, we "conciliate" the general method on one hand, with two particular "no-exotic-electric-charge" 3-3-1 family models on the other hand, looking for their exact solution. We focus our analysis on the boson mass issue. Since the fundamental triplet in the class of models we intend to tackle is a quark triplet belonging to the same representation of the local gauge group  $q_L = \begin{pmatrix} u & d & D \end{pmatrix}_L^T \sim (\mathbf{3}, \mathbf{3}, 0)$ , we obtain just the two models C and D from Ref. [9] when we choose  $\nu_0 = 0$ ,  $\nu_1 = 0$  and  $\nu_2 = -1$  respectively. The boson mass problem will be solved first by diagonalizing the mass matrix and imposing certain conditions upon its eigenvectors. Then, the charge computing in various representations will make the difference between models.

### 3.1 Higgs Sector

The Higgs sector of any 3-3-1 model consists of three Higgs triplets  $\phi^{(1)}$ ,  $\phi^{(2)}$  and  $\phi^{(3)}$ . The general parametrization of the Higgs sector (which ensures a good non-degenerate boson mass spectrum) with the property  $Tr\eta^2 = 1 - \eta_0^2$ , reads in our concrete case:

$$\eta^2 = (1 - \eta_0^2) \text{diag} \left[ \frac{1}{2}(a + b), 1 - a, \frac{1}{2}(a - b) \right] \quad (10)$$

where, for the moment,  $a$  and  $b$  are arbitrary non-vanishing real parameters. Obviously,  $\eta_0, a \in [0, 1]$ .

Bearing in mind that for any model the above parametrization in the general method should be added with the set  $(g, \theta, \nu)$ , we opt for the following versors  $\nu_0 = 0$ ,  $\nu_1 = 0$ , and  $\nu_2 = -1$  in order to obtain (using formula (7)) the correct electric charges of the fundamental multiplet. We also establish from the very beginning that the coupling constant of the model coincides with the first coupling constant of the SM. The relation between the  $\theta$  angle from our parametrization and  $\theta_W$  from SM arises from the constraint  $e = g \sin \theta_W$ , the well-known condition in SM. Therefore, computing Eq. (7) for the quark fundamental multiplet one gets  $\sin \theta = \frac{2}{\sqrt{3}} \sin \theta_W$  which will be used in obtaining the eigenvalues of the mass matrix (3). Thus, we can compute the elements of the mass matrices, according to formulae (2) and (3).

### 3.2 Boson Mass Spectrum

For the non-diagonal bosons one obtains straightforwardly from Eq. (2):  $m_W^2 = m^2 a$ ,  $m_X^2 = m^2 [1 - \frac{1}{2}(a + b)]$  and  $m_Y^2 = m^2 [1 - \frac{1}{2}(a - b)]$ , if we consider  $m^2 = g^2 \langle \phi \rangle^2 (1 - \eta_0^2)/4$ .

For the diagonal (neutral) bosons the mass matrix will take the form

$$M^2 = m^2 \begin{vmatrix} 1 - \frac{1}{2}a + \frac{1}{2}b & -\frac{1}{\sqrt{3-4s^2}} (1 - \frac{3}{2}a - \frac{1}{2}b) \\ -\frac{1}{\sqrt{3-4s^2}} (1 - \frac{3}{2}a - \frac{1}{2}b) & \frac{1}{3-4s^2} (1 + \frac{3}{2}a - \frac{3}{2}b) \end{vmatrix} \quad (11)$$

where we have made the notation  $\sin \theta_W = s$ .

Now, one has to accept that the old neutral boson  $Z$  should be an eigenvector of this mass matrix corresponding to the eigenvalue  $m_Z^2 = m_W^2 / \cos^2 \theta_W$  also established in the SM. That is, one computes  $\text{Det} [M^2 - m_Z^2 a / (1 - s^2)] = 0$ . This leads to the following unique constraint upon the parameters  $b = a \tan^2 \theta_W$ . This result is very important since it shows that a single parameter (let it be  $a$ ) in the Higgs sector is enough in order to exactly determine all the masses involved in the model. Thus, a unique mass scale has been obtained. In this parametrization, the masses of the gauge non-diagonal bosons become:  $m_W^2 = m^2 a$ ,  $m_X^2 = m^2 (1 - a/2 \cos^2 \theta_W)$  and  $m_Y^2 = m^2 [1 - a(1 - \tan^2 \theta_W)/2]$ .

Since  $Tr(M^2) = m_Z^2 + m_{Z'}^2$ , the neutral diagonal bosons will acquire the following masses:

$$m_Z^2 = \frac{m^2 a}{\cos^2 \theta_W} \quad (12)$$

$$m_{Z'}^2 = m^2 \left[ 1 + \frac{1}{3 - 4 \sin^2 \theta_W} - a \left( 1 + \frac{\tan^2 \theta_W}{3 - 4 \sin^2 \theta_W} \right) \right] \quad (13)$$

At this moment, the mass scale is just a matter of tuning the parameter  $a$  in accordance with the possible values for  $\langle \phi \rangle$ . Thus, one can play the game of recovering all the experimental values for the bosons under consideration.

### 3.3 The General Weinberg Transformation

In our concrete 3-3-1 models, assuming the above versor choice, the gWt reads

$$A_\mu^0 = A_\mu^{em} \cos \theta + (\omega_{1\cdot}^2 Z_\mu^1 + \omega_{2\cdot}^2 Z_\mu^2) \sin \theta$$

$$A_\mu^3 = \omega_{1\cdot}^1 Z_\mu^1 - \omega_{2\cdot}^1 Z_\mu^2 \quad (14)$$

$$A_\mu^8 = -A_\mu^{em} \sin \theta + (\omega_{1\cdot}^2 Z_\mu^1 + \omega_{2\cdot}^2 Z_\mu^2) \cos \theta$$

The transformation  $\omega$  reduces here to a simple rotation of angle  $\theta'$ . Bearing in mind that the neutral boson mass matrix becomes

$$M^2 = m^2 \begin{vmatrix} \left[ 1 - \frac{a}{2} \left( \frac{1-2s^2}{1-s^2} \right) \right] & -\frac{1}{\sqrt{3-4s^2}} \left[ 1 - \frac{a}{2} \left( \frac{3-2s^2}{1-s^2} \right) \right] \\ -\frac{1}{\sqrt{3-4s^2}} \left[ 1 - \frac{a}{2} \left( \frac{3-2s^2}{1-s^2} \right) \right] & \frac{1}{3-4s^2} \left[ 1 + \frac{3}{2} a \left( \frac{1-2s^2}{1-s^2} \right) \right] \end{vmatrix} \quad (15)$$

one obtains

$$\omega = \frac{1}{2\sqrt{1-\sin^2 \theta_W}} \begin{vmatrix} \sqrt{3-4\sin^2 \theta_W} & 1 \\ -1 & \sqrt{3-4\sin^2 \theta_W} \end{vmatrix} \quad (16)$$

which will lead (through gWt) to the charges of the particles. The second neutral boson ( $Z_\mu^2$  in our notation) will be the Weinberg neutral boson while  $Z_\mu^1$  is the new one of this class of models.

### 3.4 Charge Spectrum

For the fundamental multiplet, the electric charge matrix can be expressed in our parametrization - putting  $y_\rho = 0$  in Eq. (7) - as  $Q(A_\mu^{em}) = \text{Diag}(-e/3, -e/3, +2e/3)$ . The resulting matrix corresponds to both cases - models C and D in Ref. [9] - fitting the real values of the electric charges of the known quarks. The neutral charges for the fundamental quark triplet can be expressed from Eq.(8) by

$$Q(Z_\mu) = \frac{e}{\sin 2\theta_W} \left( -T_3 + \frac{3 - 4 \sin^2 \theta_W}{\sqrt{3}} T_8 \right) \quad (17)$$

$$Q(Z'_\mu) = \frac{e \sqrt{3 - 4 \sin^2 \theta_W}}{\sin 2\theta_W} \left( T_3 + \frac{1}{\sqrt{3}} T_8 \right) \quad (18)$$

The electric charges of other multiplets (of the representations  $\rho$ ) can be obtained by adding to these expressions the hypercharges of each multiplet accordingly. The neutral charges are then

$$Q^\rho(Z_\mu) = \frac{e}{\sin 2\theta_W} \left( \frac{2}{\sqrt{3}} \sin \theta_W \sqrt{3 - 4 \sin^2 \theta_W} y_\rho - T_3^\rho + \frac{3 - 4 \sin^2 \theta_W}{\sqrt{3}} T_8^\rho \right) \quad (19)$$

$$Q^\rho(Z'_\mu) = \frac{e \sqrt{3 - 4 \sin^2 \theta_W}}{\sqrt{3} \sin 2\theta_W} \left( \frac{2 \sin \theta_W}{\sqrt{3 - 4 \sin^2 \theta_W}} y_\rho + \sqrt{3} T_3^\rho + T_8^\rho \right) \quad (20)$$

Furthermore, if one computes Eqs. (7), (19), (20), and exploits the substitutions  $\alpha = \sqrt{3 - 4 \sin^2 \theta_W}$  and  $\beta = \sin 2\theta_W$ , one gets the values of the charges of the leptons in both models (Tables 1 and 2). We are surprised to discover by means of our method the same values for all leptons as in the SM [15].

Based on techniques of the general method of exactly solving gauge models with high symmetries we have obtained the exact solution of two "no-exotic-electric-charge" 3-3-1 models. At this point they are completely solved in the sense that no approximation is necessary in order to establish the mass scale and to determine the coupling coefficients. Moreover, we proved that the SM values are not spoiled in this class of 3-3-1 model's exact solution.

### 3.5 Fermion Sector

In the following, we focus on the particular fermion content of Model D. It consists of three fermion generations: three lepton generations with the same representation and three quark generations that obey different representations with respect to the gauge group of the model. Lepton triplets are color singlets and quark triplets are color triplets. Lepton families are

$$f_{\alpha L} = \begin{pmatrix} e_\alpha \\ \nu_\alpha \\ \nu_\alpha^c \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{3}^*, -1/3) \quad e_\alpha^c \sim (\mathbf{1}, \mathbf{1}, 1) \quad (21)$$

Table 1: Charges of the particles in Model D

Charge\Particle	$e, \mu, \tau$	$\nu_e, \nu_\mu, \nu_\tau$	$N_e^0, N_\mu^0, N_\tau^0$
$Q(A^{em})$	$-e$	0	0
$Q(Z)$	$-\frac{e}{\beta}(2\sin^2\theta_W - 1)$	$-\frac{e}{\beta}$	0
$Q(Z')$	$-\frac{e\alpha}{3\beta}(\frac{2\sin^2\theta_W}{\alpha^2} - 1)$	$-\frac{e\alpha}{3\beta}(\frac{2\sin^2\theta_W}{\alpha^2} - 1)$	$-\frac{2e\alpha}{3\beta}(\frac{\sin^2\theta_W}{\alpha^2} + 1)$

Table 2: Charges of the particles in Model C

Charge\Particle	$e, \mu, \tau$	$\nu_e, \nu_\mu, \nu_\tau$	$E, T, M$
$Q(A^{em})$	$-e$	0	$-e$
$Q(Z)$	$-\frac{e}{\beta}(2\sin^2\theta_W - 1)$	$-\frac{e}{\beta}$	$-\frac{e}{\beta}2\sin^2\theta_W$
$Q(Z')$	$-\frac{e\alpha}{3\beta}(\frac{4\sin^2\theta_W}{\alpha^2} + 1)$	$-\frac{e\alpha}{3\beta}(\frac{4\sin^2\theta_W}{\alpha^2} + 1)$	$-\frac{2e\alpha}{3\beta}(\frac{2\sin^2\theta_W}{\alpha^2} - 1)$

where  $e_\alpha = e, \mu, \tau$ . We mean by superscript  $c$  the charge conjugation (for details see Appendix B in [1]) and by subscript  $L$  the chiral left-handed component. Evidently, the third component of the triplet is now considered as the right-handed component of neutrino field. This assumption does not alter the previous results and it is most likely since the right-handed neutrino does not couple to the SM neutral gauge boson (see Table 1). Therefore, it had no reason to be part of the SM.

The quarks come (according to the anomaly cancellation requirement) in three distinct generations as follows:

$$Q_{iL} = \begin{vmatrix} u_i \\ d_i \\ D_i \end{vmatrix}_L \sim (\mathbf{3}, \mathbf{3}^*, 0) \quad Q = \begin{vmatrix} d \\ u \\ U \end{vmatrix}_L \sim (\mathbf{3}, \mathbf{3}^*, 1/3) \quad (22)$$

$$(d_L)^c, (d_{iL})^c \sim (\mathbf{3}^*, \mathbf{1}, 1/3) \quad (u_L)^c, (u_{iL})^c \sim (\mathbf{3}^*, \mathbf{1}, -2/3) \quad (23)$$

$$(U_L)^c \sim (\mathbf{3}^*, \mathbf{1}, -2/3) \quad (D_{iL})^c \sim (\mathbf{3}^*, \mathbf{1}, 1/3) \quad (24)$$

with  $i = 1, 2$ .

### 3.6 Yukawa Sector

For quarks and leptons - except for neutrinos - the traditional Yukawa couplings seem to be sufficient in order to get their desired masses [4] - [8].

The lepton families in 3-3-1 models under consideration here acquire their masses through the following couplings [4] - [8]:

$$-\mathcal{L}_Y = G_{\alpha\alpha} \bar{f}_{\alpha L} \rho e_{\alpha L}^c + G_{\alpha\beta} \varepsilon^{ijk} (\bar{f}_{\alpha L})_i (f_{\beta L}^c)_j (\rho^*)_k + H.c. \quad (25)$$

The Higgs triplets should transform as  $\chi, \eta \sim (\mathbf{1}, \mathbf{3}^*, -1/3)$  and  $\rho \sim (\mathbf{1}, \mathbf{3}^*, 2/3)$  in order to match the gauge invariance conditions when they couple to fermion fields in Yukawa terms Eq. (25). Their electric charge assignment leads to  $\chi = (\chi_-, \chi_0, \chi_0)^T$ ,  $\eta = (\eta_-, \eta_0, \eta_0)^T$  and  $\rho = (\rho_0, \rho_+, \rho_+)^T$ . These scalar fields acquire, when looking for the local minimum of the scalar potential in Eq. (1), the following VEVs:  $\langle \chi \rangle = (0, 0, V_\chi)^T$ ,  $\langle \eta \rangle = (0, V_\eta, 0)^T$  and  $\langle \rho \rangle = (V_\rho, 0, 0)^T$  which generate masses of the quarks and charged leptons as in [4] - [8].

According to a traditional Dirac Ld put in the pure left form (see Appendix B in Ref. [1]), one can identify the mass of the lepton as

$$m(e_\alpha) = G_{\alpha\alpha} \langle \phi^{(\rho)} \rangle \quad (26)$$

In our method each mass source corresponds to a certain parameter which multiplies the same VEV  $\langle \phi \rangle$ . The exact solution shown above demands the following relation between the two parameters  $b = a \tan^2 \theta_W$ . Thus,  $\eta$  becomes a one-parameter matrix  $\eta^2 = (1 - \eta_0^2) \text{diag} [a/2 \cos^2 \theta_W, 1 - a, a(1 - \tan^2 \theta_W)/2]$ . Under these circumstances, for each  $a$  one can estimate the alignment of VEVs just by bijective mapping  $(\chi, \rho, \eta) \rightarrow (1, 2, 3)$  while keeping unchanged the order in the parameter matrix  $\eta$ . The charged lepton mass can have, with respect to the parameter mapping, the following possible values:

- $m(e_\alpha) = G_{\alpha\alpha} \sqrt{1 - a} \langle \phi \rangle$  (case I)
- $m(e_\alpha) = G_{\alpha\alpha} \sqrt{a(1 - \tan^2 \theta_W)/2} \langle \phi \rangle$  (case II)
- $m(e_\alpha) = G_{\alpha\alpha} \sqrt{a/2 \cos^2 \theta_W} \langle \phi \rangle$  (case III).

Up to this stage each of these cases has two subcases for the choice of the remaining two scalar triplets.

According to the gauge invariance requirement, the Yukawa coupling terms for quark families are the same as those in Refs. [4] - [8] and they supply a good quark mass spectrum within the framework of exactly solving method as well.

If one introduces the more restrictive requirement of getting neutrino mass, then only two out of the six possible cases can be taken into consideration (as we will see in Sec. 4).

## 4 Neutrino Mass

It has been considered for a long time that the neutrino is a massless elementary particle that played a crucial role in the weak sector of the SM. Recently, certain evi-

dences regarding the new phenomenon of "neutrino oscillations" were found in Super-Kamiokande [16] - [18], SNO [19] - [21], KamLAND [22], K2K [23], and other neutrino experiments [24] - [30]. These suggest a small but non-zero neutrino mass, although the nature of massive neutrinos remains an open issue. Are they Dirac or Majorana particles? Definitely, they are the signature of new physics beyond SM. Therefore, some attempts to embed neutrino mass in various extensions of the SM seem justified. In 3-3-1 models these attempts gave promising results by using distinct strategies: radiative mechanisms (1 - and 2 - loop radiative corrections [31] - [33]), the Higgs triplet method in exotic models [34], or various see-saw mechanisms [35] - [37].

We propose a different method in order to provide neutrino masses using the exact solution presented above for a particular 3-3-1 model (namely model D). Neutrinos are considered here as Majorana fields and they enter naturally into the mass mixing matrix straightforwardly from the Yukawa terms after SSB took place.

#### 4.1 Neutrino Mass Matrix

In order to obtain masses for neutrinos involved in model D, one should introduce in Eq. (25) a new term  $G_{\alpha\beta L} \bar{f}_{\alpha L} S f_{\beta L}^c + H.c.$ , with  $S \sim (\mathbf{1}, \mathbf{6}, -2/3)$  a symmetric matrix constructed out of a sum of tensor products among certain Higgs multiplets in the manner  $S = \phi^{-1} (\phi^{(\eta)} \otimes \phi^{(x)} + \phi^{(x)} \otimes \phi^{(\eta)})$ . It looks like:

$$S = \phi^{-1} \begin{pmatrix} \phi_1^{(\eta)} \phi_1^{(x)} + \phi_1^{(x)} \phi_1^{(\eta)} & \phi_2^{(\eta)} \phi_1^{(x)} + \phi_2^{(x)} \phi_1^{(\eta)} & \phi_3^{(\eta)} \phi_1^{(x)} + \phi_3^{(x)} \phi_1^{(\eta)} \\ \phi_1^{(\eta)} \phi_2^{(x)} + \phi_1^{(x)} \phi_2^{(\eta)} & \phi_2^{(\eta)} \phi_2^{(x)} + \phi_2^{(x)} \phi_2^{(\eta)} & \phi_3^{(\eta)} \phi_2^{(x)} + \phi_3^{(x)} \phi_2^{(\eta)} \\ \phi_1^{(\eta)} \phi_3^{(x)} + \phi_1^{(x)} \phi_3^{(\eta)} & \phi_2^{(\eta)} \phi_3^{(x)} + \phi_2^{(x)} \phi_3^{(\eta)} & \phi_3^{(\eta)} \phi_3^{(x)} + \phi_3^{(x)} \phi_3^{(\eta)} \end{pmatrix} \quad (27)$$

and after SSB only  $\phi_2^{(\eta)}$  and  $\phi_3^{(x)}$  will survive gauge fixing. Therefore, only the positions where they meet (23 and 32 in  $\langle S \rangle$ ) will be non-zero.

$$\langle S \rangle = \frac{1}{\langle \phi \rangle} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \langle \phi^{(x)} \rangle \langle \phi^{(\eta)} \rangle \\ 0 & \langle \phi^{(\eta)} \rangle \langle \phi^{(x)} \rangle & 0 \end{pmatrix} \quad (28)$$

The neutrino mass matrix takes then the form:

$$M = 4 \begin{vmatrix} A & D & L \\ E & B & F \\ K & G & C \end{vmatrix} \frac{\langle \phi^{(\eta)} \rangle \langle \phi^{(x)} \rangle}{\langle \phi \rangle} \quad (29)$$

with the coupling constants  $A = G_{ee}$ ,  $B = G_{\mu\mu}$ ,  $C = G_{\tau\tau}$ ,  $D = G_{e\mu}$ ,  $E = G_{\mu e}$ ,  $F = G_{\mu\tau}$ ,  $G = G_{\tau\mu}$ ,  $L = G_{e\tau}$ ,  $K = G_{\tau e}$ . We notice that for the Majorana case, matrix  $M$  is a symmetric one, that is  $D = E$ ,  $F = G$ ,  $L = K$ .

At this point there are three possible situations (depending on the VEV alignment choice) which have to be separately analyzed, all of them containing the sole parameter  $a$ . Some of them will be ruled out by certain restrictive conditions imposed by phenomenological reasons.

## 4.2 Neutrino Mixing

The physical neutrino mass issue can be addressed if we consider first neutrino mixing (for details see the excellent reviews of Bilenky [38] - [40] or Mohapatra [41] and references therein). The unitary mixing matrix  $U$  (with  $U^\dagger U = 1$ ) links the gauge-flavor basis to the physical basis of massive neutrinos in the manner:

$$\nu_{\alpha L}(x) = \sum_{i=1}^3 U_{\alpha i} \nu_{iL}(x) \quad (30)$$

where  $\alpha = e, \mu, \nu$  (corresponding to neutrino gauge eigenstates), and  $i = 1, 2, 3$  (corresponding to massive physical neutrinos with masses  $m_i$ ). In the following, we consider all neutrinos as Majorana fields, *i.e.*  $\nu_{iL}^c(x) = \nu_{iL}(x)$ . This is very plausible, since neutrinoless double  $\beta$ -decay is still under experimental fire. The mass term in the Yukawa sector yields:

$$-\mathcal{L}_\nu^{mass} = \frac{1}{2} \bar{\nu}_{\alpha L} M_{\alpha\beta} \nu_{\beta L}^c + H.c \quad (31)$$

The mixing matrix  $U$  that diagonalizes the mass matrix  $U^T M U = m_{ij} \delta_j$  has in the standard parametrization the form:

$$U = \begin{vmatrix} c_2 c_3 & s_2 c_3 & s_3 e^{-i\delta} \\ -s_2 c_1 - c_2 s_1 s_3 e^{i\delta} & c_1 c_2 - s_2 s_3 s_1 e^{i\delta} & c_3 s_1 \\ s_2 s_1 - c_2 c_1 s_3 e^{i\delta} & -s_1 c_2 - s_2 s_3 c_1 e^{i\delta} & c_3 c_1 \end{vmatrix} \quad (32)$$

where we made the substitutions  $\sin \theta_{23} = s_1$ ,  $\sin \theta_{12} = s_2$ ,  $\sin \theta_{13} = s_3$ ,  $\cos \theta_{23} = c_1$ ,  $\cos \theta_{12} = c_2$ ,  $\cos \theta_{13} = c_3$  for the mixing angles, and  $\delta$  is the CP phase.

## 4.3 Restrictions on Parameter $a$

Assuming that the new neutral boson  $Z'$  has to be heavier than the Weinberg neutral boson  $Z$ , when comparing the masses of the two bosons (formulae (12) - (13)) one obtains the condition:

$$a < \frac{2(1 - \sin^2 \theta_W)}{3 - 2 \sin^2 \theta_W} \quad (33)$$

Now, if we take  $\sin^2 \theta_W = 0.23113 \pm 0.00015$  [14] we get  $a < 0.60594 \pm 0.00005$ . The first remark is that parameter  $a$  is upper bounded in all the three cases under consideration.

Important information comes also from the trace of matrix  $M$  when combining Eqs. (26) and (29):

$$Tr M = 4m(\tau) \left[ 1 + \frac{m(\mu)}{m(\tau)} + \frac{m(e)}{m(\tau)} \right] \frac{\langle \phi^{(x)} \rangle \langle \phi^{(\eta)} \rangle}{\langle \phi^{(\rho)} \rangle \langle \phi \rangle} \quad (34)$$

Bearing in mind that, at the same time,  $Tr M = \sum_i m_i$  and phenomenological values  $m_i$  of neutrino masses are severely limited to few eVs, one remains with Case I only.

Case II and Case III are ruled out since both they give  $TrM \sim \sqrt{1-a}$ . These traces reach their minimum values when parameter  $a$  reaches its maximum allowed value. Thus, these cases are not compatible with the right order of magnitude for neutrino masses that can be provided only by very small values of parameter  $a$  in Case I. Therefore, a good solution is:

$$\sum_i m_i \simeq m(\tau) \left( \frac{2a}{\sqrt{1-a}} \right) \frac{\sqrt{1-2\sin^2\theta_W}}{\cos^2\theta_W} \quad (35)$$

where we neglected the small ratios  $m(\mu)/m(\tau) \sim 0.05$  and  $m(e)/m(\tau) \sim 0.0002$  in Eq. (34).

Now, if we consider as valid the values offered by Tritium  $\beta$ -decay experiments Troitsk [42] and Mainz [43] - [44] which consider that  $m_\beta \leq 2.3$  eV, then parameter  $a$  has to be in the range  $a < 0.678 \cdot 10^{-9}$ . This implies for  $VEV < \phi > \sim 0.95 \cdot 10^6$  TeV and the resulting masses for the new bosons:  $m_X \simeq m_Y > 3.1 \cdot 10^3$  TeV and  $m_{Z'} > 3.7 \cdot 10^3$  TeV.

Based on cosmological data [45], the upper limit is even more restricted  $\sum_i m_i < (0.4 - 1.7)$  eV, which leads to  $a < (0.118 - 0.501) \cdot 10^{-9}$ . Consequently, bosons will acquire masses:  $m_X \simeq m_Y > (3.6 - 7.5) \cdot 10^3$  TeV and  $m_{Z'} > (4.4 - 9.1) \cdot 10^3$  TeV at a breaking scale  $< \phi > \sim (1.1 - 2.3) \cdot 10^6$  TeV.

While the "old" bosons remain at their SM values, the "new" ones now have to become very massive. This is the price paid in order to have good phenomenological results for all the particles in this 3-3-1 model, using only one free parameter. Notwithstanding, we note that such massive bosons do not contradict the possible mass evaluation communicated in Ref. [14] where they are accepted as  $m_X \simeq m_Y > 800$  GeV and  $m_{Z'} > 1500$  GeV. Therefore, one can consider that neutrino masses could arise (avoiding the see-saw prescriptions, the method of 1 - and 2 - loop radiative mechanisms or any other approximations), only as a consequence of a very large breaking scale of the model and accompanied by very massive bosons.

#### 4.4 Mass Squared Differences

For physical neutrinos, mass squared differences - which are experimentally accesible - are defined as  $\Delta m_{ij}^2 = m_j^2 - m_i^2$ . Their right order of magnitude can be obtained for  $\Delta m_{12}^2 \leq 8 \cdot 10^{-5}$  eV<sup>2</sup> from solar and KamLAND data [19] - [22] and for  $\Delta m_{23}^2 \leq 2 \cdot 10^{-3}$  eV<sup>2</sup> from Super Kamiokande atmospheric data [16] - [18].

Considering that in Eq. (29) the coupling constants act as variables, the diagonalization of the matrix  $M$  is equivalent to a system of 9 linear equations with 12 variables (6 linear equations with 9 variables, if  $M$  is symmeric - Majorana case) which leads to the following solution for the physical neutrino masses:

$$m_i = f_i(A, B, C) \frac{< \phi^{(x)} > < \phi^{(\eta)} >}{< \phi >} \quad (36)$$

for  $i = 1, 2, 3$ . They can be put in a more explicit form of the Case I

$$m_i = F_i \left[ \frac{m(\mu)}{m(e)}, \frac{m(\tau)}{m(e)}, \theta_{12}, \theta_{13}, \theta_{23} \right] \left( \frac{a}{\sqrt{1-a}} \right) \frac{\sqrt{1-2\sin^2\theta_W}}{2\cos^2\theta_W} m(e) \quad (37)$$

where  $F_i$  are linear combinations of mass ratios:

$$F_i \left[ \frac{m(\mu)}{m(e)}, \frac{m(\tau)}{m(e)}, \theta_{12}, \theta_{13}, \theta_{23} \right] = \alpha_i + \beta_i \frac{m(\mu)}{m(e)} + \gamma_i \frac{m(\tau)}{m(e)} \quad (38)$$

Coefficients  $\alpha_i, \beta_i, \gamma_i$  are analytical functions depending on mixing angles and they result from solving the linear system of diagonalization.

The mass squared differences are now:

$$\Delta m_{ij}^2 = (F_j^2 - F_i^2) \left( \frac{a}{\sqrt{1-a}} \right)^2 \frac{(1-2\sin^2\theta_W)}{4\cos^4\theta_W} m^2(e) \quad (39)$$

A more detailed analysis of the resulting values for mass squared differences (when LMA conditions are embedded in our method) will be presented elsewhere. Here we restrict ourselves to observing only that for  $a \sim 10^{-9} - 10^{-10}$  the necessary order of magnitude for  $(F_j^2 - F_i^2)$  has to be  $\sim 10^6$  in order to fit experimental data. This is quite plausible, assuming that in Eq. (38) the leading (larger) term is  $\gamma_i m(\tau)/m(e)$  and its order of magnitude is at most  $\sim 10^3$ .

## 5 Concluding Remarks

We have presented an original method of exploring neutrino mass on theoretical grounds, within the framework of the exact solution of a particular 3-3-1 gauge model. The unusual coupling among the lepton multiplets (tied together by tensor-like products among Higgs multiplets) led to an interesting solution in accordance with actual phenomenological data.

Concerning the general method applied here, we observe that the "geometrization" of the Higgs sector requires only one parameter (since  $\eta_0$  can be incorporated in the  $\text{VEV} < \phi >$ ). In order to investigate mass spectrum, one has to tune the sole parameter  $a$ . For instance, if  $a \simeq 1$  then the mass scale will be upper bounded by the Weinberg neutral boson's mass and the new neutral boson will gain a mass of about  $m_{Z'} \simeq m_Z/2$ . This situation is definitely ruled out by phenomenological reasons. If the more plausible  $a \simeq 0$  occurs, then the upper limit of the mass scale will be given by the new neutral boson which could stand, in principle, at any level coming from  $\text{VEV} < \phi >$ . The value can be accommodated to fit the known masses of the "old" bosons in the model. If one wants to keep the  $m_{Z'} > m_Z$  condition, then the parameter must be  $a < 0.6$ . More severe restrictions on this parameter which require the range  $a \sim 10^{-9}$  come from neutrino data.

Inputting from the beginning the set of parameters  $(g, \theta, \nu)$ , one observes for models under consideration here that the "old" charges of the particles reproduce through our method their values from SM, while the "new" charge presents indistinct values

for the SM leptons. This result makes obvious the fact that, with respect to this new boson, the SM leptons present the same behavior and that could be the sign of the new physics hidden by this new boson at energies above the TeV scale. At the same time, the charges are independent from the boson masses, also as in the SM. One can observe for Model D that the new neutral particles (right-handed neutrinos) do not couple to  $Z$ . This could explain why the right-handed neutrinos are not manifest in SM and - as a consequence - why neutrino mass had to be identical to zero in SM.

Assuming that parameter  $a < 0.6$  one can evaluate a unique mass scale for all fermions and gauge bosons in the model, encountering three possible cases for the alignment of VEVs  $\langle \phi^{(\eta)} \rangle$ ,  $\langle \phi^{(\rho)} \rangle$ ,  $\langle \phi^{(\chi)} \rangle$ . In order to keep our method flexible we eliminated cases II and III (Sec. 4) - where "the smaller the parameter, the larger the neutrino mass" - for they give an unacceptable order of magnitude for the neutrino mass when  $a \rightarrow 0$ . Only case I allows  $m(\nu_i) \ll m(e)$  and therefore it seems to be the most likely. In addition, the upper limit on the values of parameter  $a$  offers a lower limit on the VEV  $\langle \phi \rangle$ . When comparing the mass of the Weinberg neutral boson  $Z$  obtained at a scale  $\langle \phi_0 \rangle \sim 174$  GeV in the SM to the mass of the same boson obtained in our 3-3-1 model, one has to impose that these ones be identical. This requirement leads to the following condition  $\langle \phi \rangle \geq 316$  GeV which is also plausible, leading to the required larger values for the breaking scale in order to keep consistency with neutrino mass phenomenology.

In conclusion, we consider that the exact solution of the 3-3-1 models without exotic electric charges presented above can act as a viable theoretical background for the electro-weak phenomenology. We proved that it can fit all the available experimental data and predict the existence of very heavy new bosons, using only one free parameter. These are the basic requirements of a good electro-weak theory that in addition includes (at a very large breaking scale) a suitable mechanism which allows neutrinos to acquire their Majorana masses.

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